



University College Dublin  
An Coláiste Ollscoile, Baile Átha Cliath

**SPECIMEN EXAMINATION 2015/2016**

**MATH00030**

**Access to Science, Engineering and Agriculture: Mathematics 1**

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**Time Allowed: 2 hours**

**Instructions for Candidates**

Candidates should attempt all questions.

Note that not all questions are allocated the same number of marks.

**Notes for Invigilators**

Non programmable calculators are permitted.

The formula sheet provided is permitted.

1. (a) Without using a calculator, calculate the following.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.

(i)  $\frac{2}{9} - \frac{3}{7}$

(ii)  $\frac{7}{4} \times \left(-\frac{3}{8}\right)$

(iii)  $\frac{5}{8} \div \frac{8}{5}$

(iv)  $-5^2$

(v)  $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$

(vi)  $6 + 7 \times (-8) - 9$

(vii)  $\log_5 25$

(viii)  $\log_4 \frac{1}{16}$  [8]

- (b) Simplify the following expressions by expressing them as a power of  $x$  and/or  $y$ , as appropriate.

(i)  $x^5 \times x^{-2}$

(ii)  $x^{\frac{1}{4}} \div x^{-\frac{2}{5}}$

(iii)  $(x^{-2})^3$

(iv)  $(x\sqrt[4]{y})^4$  [5]

- (c) (i) Approximate 20.850 to one decimal place.

(ii) Approximate 0.0001234 to three significant figures.

(iii) Express 1234543.21 in scientific notation.

(iv) Express 0.000345 in scientific notation to two significant figures. [4]

(d) Simplify  $(3x - 2) - (3x^2 - x - 4)$ . [1]

(e) Multiply out  $(2x^3 + 3x)(x^2 - 2)$ . [2]

(f) Perform long division on  $\frac{x^2 - 3x + 1}{x + 2}$ , giving the quotient and remainder. [4]

(g) Evaluate  $\sum_{i=-1}^3 i^2$  [2]

(h) Calculate  $\binom{7}{3}$  without using a calculator.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator. [2]

(i) Expand  $(3x - y)^3$  using The Binomial Theorem. [4]

2. (a) Find the equation of the line through the point  $(1, -3)$  parallel to the line  $y = -2x + 2$ . [2]

- (b) Solve the simultaneous equations

$$3x - 4y = -5$$

$$4x - 3y = -2$$

[3]

- (c) Find the midpoint of the line segment joining  $(0, 3)$  and  $(4, -2)$ . [1]

3. (a) Write the expression  $3x^2 - 5x - 2$  in completed square form. [3]

- (b) Solve the equation  $3x^2 - 5x - 2 = 0$  by using the quadratic formula. [2]

- (c) Sketch the graph of the function  $y = 3x^2 - 5x - 2$ , showing the  $y$ -intercept, the  $x$ -intercept(s) (if applicable) and the turning point. [4]

4. (a) For each of the following:

- Say whether or not it is a function and if not say why not.
- If it is a function state the domain and the codomain.

(i)

$$f: \mathbb{R}^+ \rightarrow \mathbb{R}^-$$

$$x \mapsto -x + 2$$

(ii)

$$f: \mathbb{R} \rightarrow \mathbb{R}^-$$

$$x \mapsto x^2 - 2$$

[4]

- (b) Sketch the graph of the function

$$f: \{-4, -2, 0, 2, 4\} \rightarrow \{-1, 0, 3\}$$

$$-4 \mapsto -1$$

$$-2 \mapsto 0$$

$$0 \mapsto -1$$

$$2 \mapsto 0$$

$$4 \mapsto -1$$

[2]

(c) Figure 1 contains the graphs of four of the following functions:

(i)  $y = 2^x$

(ii)  $y = -6^x$

(iii)  $y = -\left(\frac{3}{5}\right)^x$

(iv)  $y = \log_4(x)$

(v)  $y = \left(\frac{7}{10}\right)^x$

(vi)  $y = \log_{1/3}(x)$

Match the functions to the graphs.

[4]

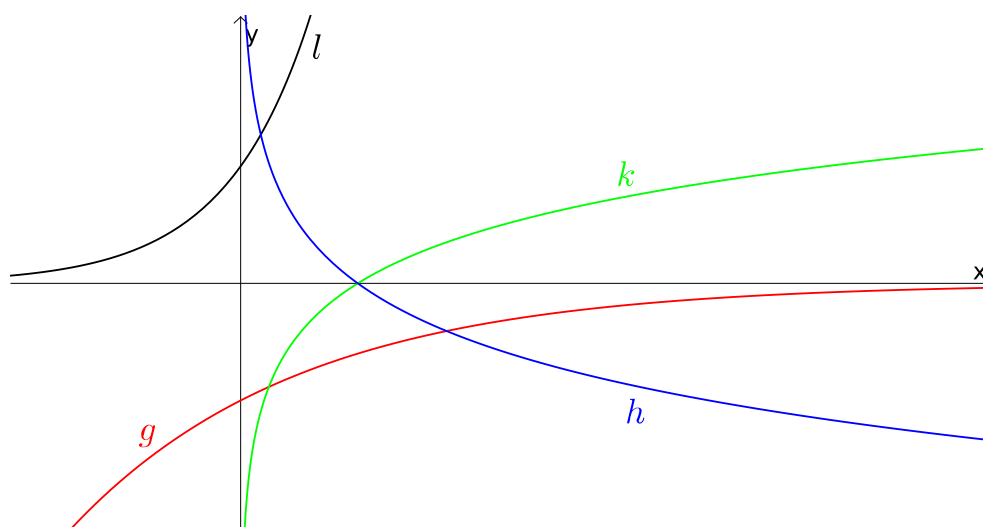


Figure 1: The functions for Question 4 (c).

(d) For each of the following functions, say whether they are injective, surjective or bijective. If a function is not injective or surjective then say why not.

(i)

$$f: \{1, 2, 3, 4\} \rightarrow \{A, B, C, D\}$$

$$1 \mapsto C$$

$$2 \mapsto D$$

$$3 \mapsto B$$

$$4 \mapsto A$$

(ii)

$$f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$$

$$x \mapsto 2x + 2$$

[3]

(e) State whether each of the functions in Part (d) has an inverse function or not, giving a reason in each case. [1]

5. (a) Convert  $285^\circ$  to radians, leaving your answer as a multiple of  $\pi$ . [1]

(b) Convert  $\frac{5\pi}{12}$  radians to degrees. [1]

(c) Using the geometric method, find  $\cos\left(\frac{4\pi}{3}\right)$  without using a calculator. [4]

(d) Using whichever trigonometric formulae you like, but without using a calculator, calculate the following.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.

(i)  $\sin\left(-\frac{\pi}{12}\right)$

(ii)  $\tan\left(\frac{5\pi}{3}\right)$  [4]

(e) Find the length of the side  $a$  in the triangle in Figure 2. [3]

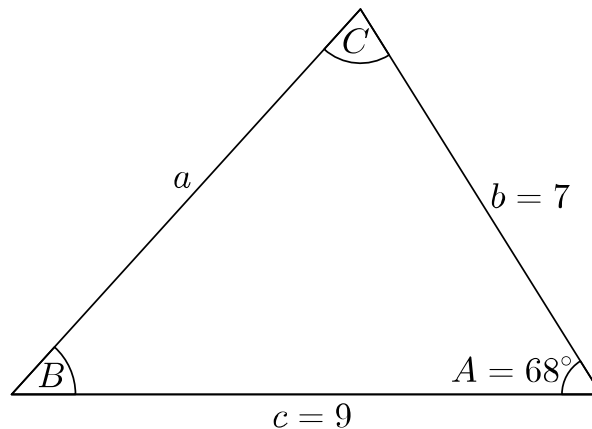


Figure 2: The triangle for Question 5 (e).

6. (a) Find the derivative of  $f(x) = 2x^2$  using first principles. [3]

(b) Find the derivatives of the following functions.

(i)  $f(x) = \sin(3)$

(ii)  $f(x) = x^5$

(iii)  $f(x) = \sin(-4x)$

(iv)  $f(x) = \cos(-3x)$

(v)  $f(x) = 3x^{\frac{1}{3}} + 2e^{-3x} - 3\ln(-3x)$  (where  $x < 0$ ) [6]

7. Find the following integrals.

(a)  $\int 4 dx$  [1]

(b)  $\int_1^2 x^7 dx$  [2]

(c)  $\int_0^{\frac{\pi}{2}} \cos(-2x) dx$  [2]

(d)  $\int e^{4x} + 3x^{-\frac{2}{3}} dx$  [2]

8. (a) For the list of numbers 2, 3, -9, -5, 7, 5, 1, find the

(i) Mean

(ii) Median

(iii) Mode(s)

(iv) Interquartile range [5]

(b) Find the line of best fit using the least squares method with the points

$(-1, -1), (0, 0), (1, 1), (4, 2)$  and  $(5, 3)$ . [8]

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